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Introduction

This project started as I observed how my son was learning numbers in school. I was astonished to realize that he was learning very much the same way I learnt many years ago; same difficulties, same limitations and same misunderstandings.

To begin with, he was learning to count by heart; not really understanding what he was doing and, of course, not knowing what counting was for. He did not understand what number was bigger and did not grasp the notion of successor.

Things got a bit worse when he started to learn how to add / subtract numbers. He was learning the operations, counted with his fingers and had difficulties getting what it was for as soon as the problems demanded some extra thinking and were not a straightforward addition / subtraction operation.

I began to think if I could find a simple way to make him “see” what he was doing.

Definition of the problem

I was sure that his fingers were very helpful because he could feel what he was doing. I observed that was probably the power of fingers: the metaphor and their physicality.

The metaphor because they can be anything that has to be counted, added and subtracted; apples, pears, horses, legs, etc. Unlike numerals –which are abstract symbols– fingers can work closer to a referent –they become indexes or indicators. Numerals, on the opposite side, have not one single clue to indicate their meaning.

Their physicality, because children can see and feel them. They see the series, they experience that 3 comes after 2 because you add one to the ancestor. If they add quantities, they use more fingers, if they subtract, they hide them. At the same time, fingers are very limited. Not only we can only use a maximum of ten, it is also that their advantages end up right where we mentioned above: count, add and subtract.

It seemed clear that these were key factors to make things simpler.

Nothing in the form of number '3', in the significant, that shows or helps to understand its meaning. But for the Romans, it was 'III' which is a big difference: its significant is an indication of its meaning. But the roman numerals have their limitations too because it gets more complicated when the position of the character changes the value of the number. For example, XI is not the same as IX. It is also very difficult to handle and understand big quantities.

The problem was clear at this point. I wanted to design a system that could help understand not only quantities, also operations and properties of numbers to help children understand them. Since the audience has peculiarities, it would be an improvement if the system was visual, physical, low-cost and, why not, a game to have fun while learning.

Elements of the problem

Following –one of my heroes– Bruno Munari's methodology I divided the problem into its constituent parts or elements. I came out with a list of requirements for my project:

- To understand that when children are counting they are adding one to the antecessor
- That a number can be the result of different operations on different numbers. This is, $5=3+2=4+1=7-2=10/2$, etc.
- Commutative property: $4+3=3+4$
- To help understand adding, subtracting, dividing, multiplications, etc. Not the algorithmic operations, but the meaning and the utility of them. Help them to know when to use them and for what kind of problems
- Divisibility
- Negative numbers
- Fractions
- Powers of two or squared numbers. What they are and what kind of problems do they

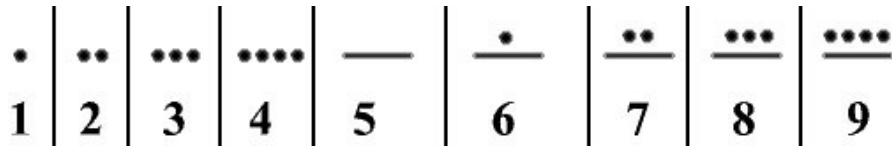
solve

- Above all, something that would make them see numbers as something physical, useful, playful, funny, that could also be extended and cost effective.

Data & information

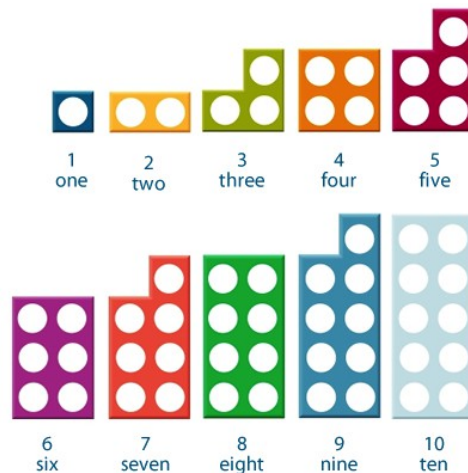
I started to look at some references; from primitive cultures to modern toys. Among the most interesting ones I found, I will describe two:

- Numerals from Maya people. I searched for greek, aramaic, and other cultures numerals but the ones that called my attention were these:



Simple, very graphic and very clear. Adding and subtracting is straightforward, but number five, a long line raises a level of complexity too difficult for children.

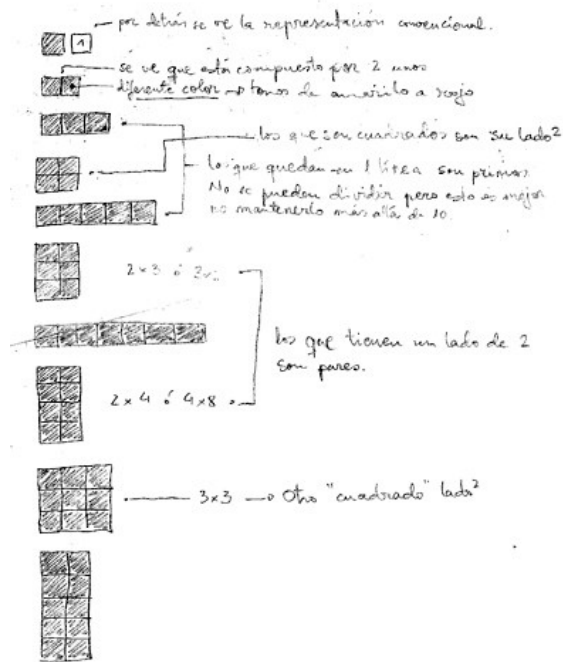
- Numicon. All modern toys/systems I found, concentrated just on representing quantity. Numicon goes a bit further because they have a few features that were interesting for me:
 - First of all, the holes are very appropriate for children because they can put their fingers inside to help them counting each piece.
 - The shape of the pieces was also very well thought because understanding odd and even numbers is very easy. Odd numbers are the ones that have one “extra” hole.



But Numicon makes subtracting a bit strange because there are not “negative pieces”. I also was looking for something more elaborate: fractions, powers, divisibility.

Creativity and sketches

Anyway, when I found out about Numicon I had already made my sketches that I show below:

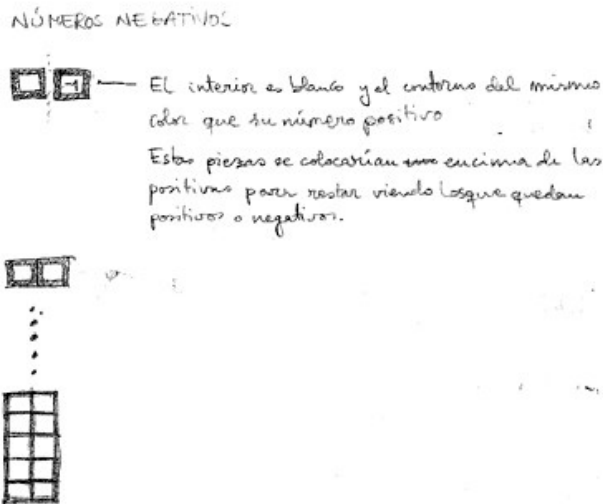


* Con números se ve mejor que cada número es el anterior + 1
pero si saben contar no es tan importante...

💡 Se ve mecanismos de empuje parecido a los puntos.
porque siempre es igual. Idealmente que se puedan ver
los números...

You can see that 1, 2, 3, 5 and 7 are prime numbers because they just have one row. Numbers with more than one row/column are divisible by the numbers in that row/column. Number nine is divisible by 3 but it can be seen that it is also 3 squared (because of its shape), etc.

I added negative numbers easily:



Hiding the positive numbers with the negative ones would make subtraction very intuitive even when the result is negative (!)

Powers of two are formed, appropriately, creating a squared shape combination.

And then I elaborated a bit more to include $\frac{1}{2}$ and $\frac{1}{4}$ in a very straightforward way because I just had to cut number one in two or four pieces.

Finally, in order to make it more fun when children are alone I added a gameboard, questions and rules so that children could play together and/or with their parents.

POTENCIAS AL CUADRADO

Formar un cuadrado que tenga por lado el número a elevar

$$3^2 \rightarrow \begin{array}{|c|c|c|} \hline \text{X} & \text{X} & \text{X} \\ \hline \text{X} & \text{X} & \text{X} \\ \hline \text{X} & \text{X} & \text{X} \\ \hline \end{array} \quad 4^2 \rightarrow \begin{array}{|c|c|c|c|} \hline \text{X} & \text{X} & \text{X} & \text{X} \\ \hline \text{X} & \text{X} & \text{X} & \text{X} \\ \hline \text{X} & \text{X} & \text{X} & \text{X} \\ \hline \text{X} & \text{X} & \text{X} & \text{X} \\ \hline \end{array}$$

POTENCIAS AL CUBO Y SUPERIORES..

$$2^3 \rightarrow \begin{array}{|c|c|c|} \hline \text{X} & \text{X} & \text{X} \\ \hline \text{X} & \text{X} & \text{X} \\ \hline \text{X} & \text{X} & \text{X} \\ \hline \end{array} = \begin{array}{|c|c|} \hline \text{X} & \text{X} \\ \hline \text{X} & \text{X} \\ \hline \end{array} + \begin{array}{|c|c|} \hline \text{X} & \text{X} \\ \hline \text{X} & \text{X} \\ \hline \end{array} + \begin{array}{|c|c|} \hline \text{X} & \text{X} \\ \hline \text{X} & \text{X} \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \text{X} & \text{X} & \text{X} \\ \hline \text{X} & \text{X} & \text{X} \\ \hline \text{X} & \text{X} & \text{X} \\ \hline \end{array}$$

$3 \times 3 = 9$ → $9 \times 3 = 27$

↑
Descomponerlo en multiplicaciones.

FRACCIONES

Proporcionalmente piezas más pequeñas!

$$\begin{array}{l} \square = 1/2 \\ \square = 1/4 \\ \square = 1/8 \end{array} \quad \begin{array}{l} \square = 1/3 \\ \square = 1/6 \\ \square = 1/12 \end{array}$$

Tests and experimentation

I tried the system with my child and his friends and the results were impressive:

- For example, he had had difficulties understanding that $4+3 = 3+4$ but with my system, this proposition was evident.
- If I told them to create number eight in as many ways as they could, they would think that when they did it with eight pieces of one, the result (8) was bigger than when made with two pieces of 4 because they were using less pieces. The misunderstanding was cleared when I joined the pieces of one as if they were two pieces of four. They saw the equality straight ahead.
- Children aged 4 or 5 were able to solve multiplication problems without help.
- Relationships like greater than and less than were also intuitive.
- Odd and even numbers were explained telling children that even numbers were formed joining two identical numbers together.
- Children would also use the pieces to create figures and other games out of their rules and imagination.
- It was also used to explain other magnitudes like seconds, milimeters, days, etc.
- Multiplication tables and series were intuitive
- I have been reported by other parents that the system was successfully used to explain percentages and proportions to fourteen years old children.

With the priceless help and comments of my dear friend, Misha Zeitlin, I designed a new version in English that improved the cutting out process and made questions clearer and more comprehensible to children.